(6.6) Solving Word Problems in Vertex and Standard Form

When solving word problems involving a quadratic equation examine the form the equation is in and decide upon the most appropriate method to answer the question.

STANDARD FORM - **Want VERTEX (maximum or minimum)**

1. Complete the square (if no fractions) to put the equation in vertex form
2. Find the zeros (factoring or quadratic formula) then find the vertex using midpoint and substitution
3. Use $x = \frac{-b}{2a}$ and substitute into the equation to find y.

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STANDARD FORM - **Want ZEROS**

1. Factor, if possible
2. Use the quadratic formula

VERTEX FORM - **Want ZEROS**

1. Expand the equation into standard form and factor, if possible
2.. Expand the equation into standard form and use quadratic formula

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Example 1: A golf ball is hit off the top of a cliff. Its height, h, in meters, after t seconds is given by

$h = -5(t - 1.2)^2 + 72.2$

a) How high is the cliff?

b) After how many seconds does the ball hit the ground? Since we have the equation in both standard and vertex form we can use either method.

c) What is the maximum height the golf ball reaches?

d) When does it reach its maximum height?

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Example 1: A golf ball is hit off the top of a cliff. Its height, h, in meters, after t seconds is given by

$h = -5(t - 1.2)^2 + 72.2$

a) How high is the cliff? $t = 0$

$h = -5(0 - 1.2)^2 + 72.2$
$h = -5(-1.2)^2 + 72.2$
$h = -5(1.44) + 72.2$
$h = -7.2 + 72.2$

$\rightarrow$ 65 m

The cliff is 65m tall.

b) After how many seconds does the ball hit the ground? Since we have the equation in both standard and vertex form we can use either method.

$h = -5(t - 1.2)^2 + 72.2$
$h = -5(t^2 - 2.4t + 1.44) + 72.2$
$h = -5t^2 + 12t - 7.2 + 72.2$
$h = -5t^2 + 12t + 65$

$0 = -(5t^2 - 12t - 65)$

$a = +5$
$b = -12$
$c = -65$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(+5)(-65)}}{2(+5)}$

$= \frac{+12 \pm \sqrt{144 + 1300}}{10}$

$= \frac{+12 \pm \sqrt{1444}}{10}$

$= \frac{12 \pm 38}{10}$

The ball hits the ground at 5s.

$\frac{12+38}{10}$, $\frac{12-38}{10}$

5s, $\cancel{-2.65}$

c) What is the maximum height the golf ball reaches?

At 1.2sec the ball reaches a maximum height of 72.2m.

d) When does it reach its maximum height?

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Example 2: Christina has 160m of edging to enclose a rectangular area to form a garden. The possible areas of the garden can be modelled by the relation $A = -w^2 + 80w$, where A is the area in square metres and w is the width of the garden in metres.
What width(s), to the nearest tenth of a metre, will generate a garden with an area of $1007 m^2$?

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What width(s), to the nearest tenth of a metre, will generate a garden with an area of $1007 m^2$?

$A = -w^2 + 80w$

$1007 = -w^2 + 80w$

$0 = -w^2 + 80w - 1007$

$0 = -(w^2 - 80w + 1007)$

$a = 1$
$b = -80$
$c = +1007$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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What width(s), to the nearest tenth of a metre, will generate a garden with an area of $1007m^2$?

$1007 = -w^2 + 80w$

$0 = -w^2 + 80w - 1007$

A(M

$0 = -[w^2 - 80w + 1007]$

a = 1
b = -80
c = +1007

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{(-80) \pm \sqrt{(-80)^2 - 4(1)(1007)}}{2(1)}$

$\frac{+80 \pm \sqrt{6400 - 4028}}{2}$

$\frac{+80 \pm \sqrt{2372}}{2}$

$\frac{+80 \pm 48.7}{2}$

$\frac{80 + 48.7}{2}$, $\frac{80 - 48.7}{2}$

$\frac{128.7}{2}$ $\frac{31.3}{2}$

64.4 15.7

15.7 $\boxed{1007m^2}$ 64.4 $\boxed{31.3/2}$

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Example 3: A crystal manufacturer that makes bowls has a daily production cost C, in dollars given by the relation $C = 2b^2 - 100b - 800$, where b is the number of bowls made.

a) How many bowls should be made to minimize the daily production cost?

b) What is the production cost when this many bowls are made?

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Example 3: A crystal manufacturer that makes bowls has a daily production cost C, in dollars given by the relation $C = 2b^2 - 100b - 800$, where b is the number of bowls made.

a) How many bowls should be made to minimize the daily production cost?

$C = 2b^2 - 100b - 800$

$C = 2[b^2 - 50b] - 800$ $\left(\frac{b}{2}\right)^2$ $\left(\frac{50}{2}\right)^2$ 625

$C = 2[b^2 - 50b + 625 - 625] - 800$

$C = 2[(b-25)^2 - 625] - 800$

$C = 2(b-25)^2 - 1250 - 800$

$C = 2(b-25)^2 - 2050$ $(25, -2050)$

b) What is the production cost when this many bowls are made?

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Example 4: An ice cream vendor at the Tulip Festival says the relation between his profit and the number of ice cream cones he sells is $P = -5x^2 + 35x - 35$, where P is his profit, in hundreds of dollars and x is the number of cones he sells, in thousands. Determine the maximum profit he can make during the Tulip Festival.

vertex

P = profit in hundreds of $
x = cones in thousands

COMPLETE THE SQUARE

$P = -5(x^2 - 7x) - 35$ $\left(\frac{b}{2}\right)^2$

$P = -5(x^2 - 7x + \frac{49}{4} - \frac{49}{4}) - 35$ $\left(\frac{7}{2}\right)^2$

$P = -5[(x - \frac{7}{2})^2 - \frac{49}{4}] - 35$ $49/4$

$P = -5(x - \frac{7}{2})^2 + \frac{245}{4} - 35$

$P = -5(x - 3.5)^2 + 61.25 - 35$

$P = -5(x - 3.5)^2 + 26.25$

$(3.5, 26.25)$

When I sell 3500 cones I maximize profit at $2625.

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Homework

Pg. 357 # 2, 3, 6, 8, 9, 10, 14
& Handout

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1. A baseball is hit and its height is given by the relation $h = -5t^2 + 20t$, where h is the height in metres and t is the time in seconds.
   a) What is the maximum height reached by the ball? **[20 m]**
   b) When does the ball reach its maximum height? **[2 s]**
   c) How long is it in the air? **[4 s]**

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2. Greg is tossing pine cones from the side of an old quarry. The pine cones fall into the water-filled hole below. The height, h, in metres of the pine cone above the surface of the water is approximately given by $h = -5t^2 + 15t + 20$, where t is time in seconds since Greg tossed the pine cone.
   a) How long did it take for the pine cone to hit the water? **[4 s]**
   b) How high is Greg standing from the water? **[20 m]**
   c) What is the height of the pine cone after 1.5 seconds? **[31.25 m]**

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3. A computer software company models the profit on its latest game using the relation $P = -2x^2 + 28x - 66$, where x is the number of games it produces in hundred thousands and P is the profit in millions of dollars.
   a) What is the maximum profit the company can earn? **[$32,000,000]**
   b) How many games must it produce to earn this profit? **[700,000]**
   c) The company breaks even when there is neither a profit nor a loss (profit = 0). What are the break-even points for the company? **[300,000 & 1,100,000 games]**

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4. A model rocket is shot straight up from the roof of the school. The height at any time is approximated by the model $h = -5t^2 + 20t + 25$, where h is the height in metres and t is the time in seconds.
   a) What is the height of the school? **[25 m]**
   b) How long is the rocket above a height of 40 m? **[2 s]**
   c) When does the rocket hit the ground? **[5 s]**
   d) What is the maximum height of the rocket? **[45 m]**

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5. The Wheely Fast Co. Makes custom skateboards for professional riders. They model their profit with the function $P = -2b^2 + 14b - 20$, where b is the number of skateboards they produce, in thousands, and P is the company's profit in hundreds of thousands of dollars.
   a) At what production level(s) does Wheely Fast Break even? **[2,000 & 5,000 skateboards]**
   b) How many skateboards does Wheely Fast need to produce to maximize profit? **[3,500]**

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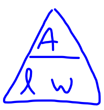
6. Farmer Brown determines that the expression $A = -3w^2 + 48w$ models the area of a rectangular garden, where A represents area in square metres and w represents the width of the garden in metres. What dimensions produce and area of $180m^2$? **[10 m by 18 m; 6 m by 30 m]**

$A = -3w^2 + 48w$



$180 = -3w^2 + 48w$

$0 = -3w^2 + 48w - 180$

$0 = -3(w^2 - 16w + 60)$

A | M
-16 | +60
-6 -10

$0 = -3[w^2 - 6w - 10w + 60)$

$0 = 3\ w(w-6) - 10(w-6)$

$0 = -3(w-6)(w-10)$

(6, 180) (10, 180)

30

6m $\boxed{180}$

18

10m $\boxed{180}$

7. Fashion Fun Company determined that the relation $R = (10 + 2x)(15 - 3x)$ modelled expected revenue for Sacred Heart sweat pants. R is revenue in dollars, and x is the amount of the change in price. Determine the change in price that will result in revenue of $126. **[increase of $2 ; decrease of $2]**

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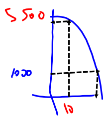
8. A pair of skydivers jump out of an airplane 5.5 km above the ground. The relation $h = 5500 - 5t^2$ is an approximate model for the divers' altitude in metres at t seconds after jumping out of the plane.
   a) After 10 s how far have the divers fallen? **[500 m]**
   b) They open their chutes at an altitude of 1000 m. How long did they free-fall? **[30 s]**



$h(t) = -5t^2 + 5500$

a) 10s $h(10) = -5(10)^2 + 5500$

$= -5(100) + 5500$

$= -500 + 5500$

$h(10) = 5000$

The diver fell 500 m.

b) $h = -5t^2 + 5500$

$ax^2 + bx + c$

$1000 = -5t^2 + 5500$

$0 = -5t^2 + 5500 - 1000$

a = +1
b = 0
c = -900

$0 = -5t^2 + 4500$

$0 = -5(t^2 - 900)$ ← or quad formula

$0 = -5(t - 30)(t + 30)$ difference of squares

30 $\cancel{-30}$ -ve time

The skydiver reaches 1000m at 30 sec.

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(6.6) Solving Problems Using Quadratic Equations

The **solutions** to a quadratic equation are called the **roots** or **zeros** or **x-intercepts**.

To find the solution/zeros the quadratic equation must equal zero.
Then, factor to find the zeros.

Example 1: Solve $h=-3t^2+6t+24$

$h=-3(t^2-2t-8)$

$h=-3(t+2)(t-4)$

A | M
-2 | -8
+2, -4

When solving word problems involving a quadratic equation examine the form the equation is in and decide upon the most appropriate method to answer the question.

Example 2: Solve $-4x=-x^2+21$

$+x^2-4x-21=0$

$(x+3)(x-7)=0$

-3 $+7$

A | M
-4 | -21
1, 21
+3, -7

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Example 3: Mike hits a golf ball upwards from the top of a cliff. The height of the ball above the base of the cliff is approximated by the model $h=-6t^2+24t+72$ where h is height in metres and t is time in seconds.

a) How high is the cliff? $t=0$

∴ the cliff is 72m high

b) How long does it take for the ball to reach a height of 42 m?

A | M
-24 | -5

$42=-6t^2+24t+72$

$0=-6t^2+24t+72-42$

$0=-6t^2+24t-30$

$0=-6(t^2-4t-5)$

$0=-6(t-5)(t+1)$

the ball reaches 42 m at 5 sec.

c) How long does it take for the ball to reach the ground?

$0=-6t^2+24t+72$

d) What is the maximum height of the ball?

e) How long is the ball above a height of 90 m?

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May 2-7:18 AM